

STABILIZATION OF CHROMOMAGNETIC FIELDS AT HIGH TEMPERATURE?

DAVID PERSSON

*Department of Physics and Astronomy
The University of British Columbia
Vancouver, B.C. V6T 1Z1, Canada
E-mail: persson@theory.physics.ubc.ca*

It is well known that a tachyonic mode appears in the spectrum of Yang—Mills theory with a static uniform magnetic field, and that the free energy has an (unstable) minimum at finite magnetic field. It is argued that spontaneous generation of magnetic field does not take place at high temperature due to nonperturbative magnetic screening. Furthermore, the dispersion relation for gauge field fluctuations in an external magnetic field at high temperature is solved. The lowest energy mode is stable against spontaneous generation of magnetic fields since it acquires a thermal mass. However, the resummed free energy (by necessity computed in the imaginary time formalism) still shows an instability, unaffected by the resummation, since the self-energy is vanishing at static Matsubara frequency.

1 The Instability and how it is Screened at High Temperature

For non-abelian gauge theories there are classes of gauge equivalent potentials corresponding to the same field strength. We shall in this letter only consider the abelian like class, that is the only one reasonable in an early universe scenario, and for simplicity only $SU(N = 2)$. Let us therefore consider a static uniform (chromo-) magnetic field in z direction in space, with the potential $A_\mu^a = \delta^{a3}(0, 0, -Bx, 0)$, where the field points in the 3-direction in group space. The $SU(2)$ Lagrangian with this background field is then rewritten in terms of the charged Vector Field $W_\mu = (W_\mu^1 + iW_\mu^2)/\sqrt{2}$. The energy spectrum reads

$$E^2(k_z, l, \sigma) = k_z^2 + (2l + 1)gB - 2\sigma gB \quad , \quad (1)$$

where the term $(2l + 1)gB$, $l = 0, 1, 2, \dots$, comes from the orbital angular momentum and the term $2\sigma gB$, $\sigma = \pm 1$, is the spin energy. The momentum parallel to B is k_z . Obviously, we have an instability in the lowest Landau level (LLL), $l = 0$, $\sigma = 1$. At $T = 0$, this leads to a spontaneous generation of a magnetic field ¹ $gB_0 = \Lambda^2 = \lambda^2 \exp\left(-\frac{48\pi^2}{11Ng^2(\lambda)}\right)$, where λ is the renormalization scale, and $g^2(\lambda)$ is running so that Λ is independent of λ . However, the free energy acquires an imaginary part, so this minimum is unstable ², A possible ground-state has to be varying over the non-perturbative scale $1/\Lambda$ —“Copenhagen Vacuum” ³.

At high temperature, we have for an asymptotically free theory the following hierarchy of scales (in due order)

T : The temperature is the typical energy of particles in the plasma and the inter-particle distance is $\sim 1/T$.

gT : The interaction of soft particles ($p \sim gT$) with hard particles ($p \sim T$) generates a thermal mass of order gT . Static electric (but not magnetic) fields are screened over the length scale $1/gT$.

g^2T : On this momentum scale Yang-Mills theory becomes non-perturbative. Theoretical arguments and lattice simulations show that non-abelian magnetic fields are screened over the length scale $1/g^2T$.

Λ : The strong coupling scale below which the vacuum theory becomes non-perturbative.

Due to the non-perturbative magnetic screening the extension of the LLL is much larger than the length-scale over which the magnetic field can be constant $1/\Lambda \gg 1/g^2T$. The would be unstable mode thus will not see a uniform field, and the Saviddy mechanism for spontaneous generation of magnetic field cannot operate.

2 External Chromomagnetic Fields

If we instead assume an external magnetic field, generated by some other mechanism, we may consider the hierarchy of scales:

$$T^2 \gg \mathcal{M}^2 \equiv \frac{N}{9}(gT)^2 \gg gB \gg (g^2T)^2 \quad . \quad (2)$$

In order to investigate if the instability is screened at high temperature, we now need to consider the dispersion relation obtained from the effective Lagrangian

$$\int d^4x d^4x' W_\mu^\dagger(\kappa, x) [-\delta(x - x') (g^{\mu\nu} D^2 - D^\mu D^\nu - 2igF^{\mu\nu}) - \Pi^{\mu\nu}(x, x')] W_\nu(\kappa', x') \quad , \quad (3)$$

where Π is the gluon self-energy in a magnetic field at high temperature. With wave-functions corresponding to the LLL, unstable at $T = 0$, the dispersion relation reads in momentum space

$$k_0^2 + gB - k_z^2 + \Pi_{\text{LLL}}(k_0, k_z) = 0 \quad , \quad (4)$$

where we integrate over perpendicular momenta

$$\Pi_{\text{LLL}}(k_0, k_z) = \int_0^\infty \frac{2p_\perp dp_\perp}{gB} e^{-p_\perp^2/gB} w_\mu^{\text{LLL}} \Pi_{\text{HTL}}^{\mu\nu}(k_0, k_z, p_\perp) w_\nu^{\text{LLL}} \quad , \quad (5)$$

and w_μ is the polarization vector in LLL. It turns out that the leading high temperature correction comes only from the ordinary hard thermal loop approximation of the gluon self-energy tensor, conveniently separated in its longitudinal (Π_{L}), and transverse (Π_{T}) parts. To leading order, the magnetic field only enters through the external states being Landau levels. In order to investigate the instability, let us consider $k_z = 0$. For $k_0 \sim \mathcal{M} \gg \sqrt{gB}$, we find to leading order

$$k_0^2 + gB - \mathcal{M}^2 \left(1 + \frac{2gB}{5\mathcal{M}^2} \right) = 0 \quad , \quad (6)$$

i.e. $k_0 \simeq \mathcal{M}(1 - 3gB/10\mathcal{M}^2)$, a stable solution. However, if we instead expand for small k_0 , we get to leading order

$$k_0^2 + gB + i \frac{3\pi^{3/2}}{8} \frac{k_0 \mathcal{M}^2}{\sqrt{gB}} = 0 \quad , \quad (7)$$

with the leading solution $k_0 \simeq i16/(3\pi^{3/2})(gB)^{3/2}/\mathcal{M}^2$. This may thus be the signal of an instability. In Figure 1, we show the two branches obtained by solving the real part of the dispersion relation for real k_0 , and then determine the imaginary part on this solution. Furthermore, the spectral weight of the stable mode is shown. For weak magnetic fields it is close to unity, leaving no phase space for the unstable mode. However, work in progress shows that when considering the full dispersion relation, there is a branch with purely imaginary k_0 , indicating the survival of the instability at high temperatures.

3 The Resummed Free Energy

In order to avoid the tree level instability, it is necessary to consider the resummed free energy, including the leading self-energy term, as well as the corresponding counter term. However, this corresponds to a sum over diagrams with vanishing momenta on the external lines (here rather no external lines), so that the real time formalism is not straightforwardly applicable⁴. Let us neglect the spatial momenta, and first consider the self-energy like function in imaginary time

$$\Theta_{\text{IT}} = -T \sum_n \Delta(i\omega_n, k) = T \sum_n \frac{1}{\omega_n^2 + k^2 + \Pi(i\omega_n, k)} \quad ; \quad \omega_n = 2\pi nT \quad , \quad (8)$$

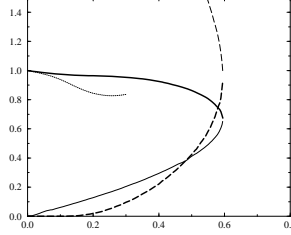


Figure 1: The solid lines show the two branches of the solution to the dispersion relation for $k_z = 0$ (fat—stable, thin—unstable). The dashed lines show the corresponding imaginary parts, and the dotted line is the spectral weight of the stable mode.

where the self-energy Π , has a branch cut along the imaginary axis. This equals the real time expression

$$\Theta_{\text{IT}} = \Theta_{\text{RT}} = \int \frac{dk_0}{2\pi} \varrho(k_0) \left\{ \frac{1}{2} + f_B(k_0) \right\} ; \quad f_B(k_0) = \frac{1}{e^{\beta|k_0|} - 1} \quad , \quad (9)$$

if we only have poles in Δ along the real axis. The spectral density is

$$-i\varepsilon(k_0)\varrho(k_0) = \text{Disc}\Delta(k_0) = \Delta(k_0 + i\epsilon) - \Delta(k_0 - i\epsilon) , \epsilon \rightarrow 0^+ \quad , \quad (10)$$

and ε is the sign function. From this we may derive that the partition function like quantity

$$\Psi_{\text{IT}} = -\frac{T}{2} \sum_n \ln \{ \beta^2 [\omega_n^2 + k^2 + \Pi(i\omega_n, k)] \} \quad , \quad (11)$$

rewritten as an integral over real energies, takes the form

$$\begin{aligned} \Psi_{\text{IT}} = & -\frac{T}{2} \int \frac{dk_0}{2\pi} \left\{ \frac{\beta|k_0|}{2} + \ln \left(1 - e^{-\beta|k_0|} \right) \right\} 2|k_0| [(1 - \text{Re } \nu) \varrho \\ & - i \text{Re } \Delta \text{Disc } \nu] - T \ln (1 - e^{-\beta k}) \quad . \end{aligned} \quad (12)$$

Obviously this differs from the corresponding real-time expression, that thus has to be wrong. It is thus necessary to compute the resummed free energy in the imaginary time formalism. The contribution from LLL reads

$$\frac{1}{\beta V} \ln Z_{\text{LLL}} = -\frac{gB}{(2\pi)^2} \int dk_z \sum_n \ln \{ \beta^2 [\omega_n^2 + k_z^2 - gB - \Pi_{\text{LLL}}(k_0, k_z)] \} \quad . \quad (13)$$

The most infra-red sensitive part is for static Matsubara frequency $i\omega_{n=0} = 0$, in which case the self-energy reads

$$\Pi_{\text{LLL}}(k_0 = 0, k_z) = 0 \quad . \quad (14)$$

When computing the free energy, the instability thus appears to be unaffected by the resummation. The instability may be removed by introducing a non-vanishing Polyakov loop⁵. This essentially amounts to replacing $\omega_n \rightarrow \omega_n - \phi/\beta$, and ϕ is determined by minimizing the free energy. Confer the recent studies of an external magnetic field on the lattice⁶, that did not show any signs of the unstable phase, and the final remarks in Section 2. It is the present author's definite opinion that this subject requires further investigation.

Acknowledgments

The first two thirds of this work was done in cooperation with Per Elmfors, as reported in Ref.⁷, to which we refer for further details and references. I would like to thank the organisers of SEWM '98. Financial support was obtained from the Nordic project "Hot Non-Perturbative Particle Physics"; and NorFA under grant no. 96.15.053-O. The author's research was funded by STINT under contract 97/121.

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